PHYSICS 630

HOMEWORK #1 **Due:** September 9, '04

Problems

1] Consider a stationary electron struck by a low-frequency EM wave, we found that the *cross-section* for this had to be

$$\sigma \sim \frac{e^4}{m^2},$$

where e is the charge and m is the mass of the electron. Use this to estimate the classical radius (r_e) of the electron in units of femtometers.

2] A function which obeys $\ddot{f} = v^2 \nabla^2 f$, describes a wave traveling with velocity v. Take the curl of Faraday's law, use a famous vector identity, consider the region to be devoid of any charge ($j_\alpha = 0$), and then relate it to the Ampere-Maxwell equation to find the propagation velocity of the electric-field wave (in the vacuum).

3] The *Compton wavelength* of a particle is taken to be the inverse of its mass. Show then that the Compton wavelength of the electron has to be given by

$$\lambda \sim \frac{r_e}{\alpha},$$

where α is the fine-structure constant.

4] To see why nobody worries about charge/current conservation when using Maxwell's field equations, start with Maxwell's equations and derive the charge continuity equation:

$$\dot{\rho} + \nabla \cdot \mathbf{J} = 0,$$

note that ∇ always means $\vec{\nabla}_x$ (the vector differential operator that acts on $\mathbf{x}$). Do not assume a vanishing magnetic field, do not use ANY vector formulas/identities (inside front or back covers of Jackson), derive everything. Next assume that the magnetic field obeys a *Clifford-type* algebra: that is, the order of the partial derivatives does matter (*i.e,* $H_{x,yz} \neq H_{x,zy}$). That being the case, derive the new charge continuity equation.